Chapter 2.5: Zeros of Polynomial Functions

Rational Zero(Root) Theorem:

If f is a polynomial and p is the factors of the constant term and q is the factors of the leading term, then the possible zeros are in the form p/q

$f(x) = qx^3 + x^2 + x + p$

$\pm \frac{P}{Q}$

List the possible zeros for:

$$f(x) = -x^4 + 3x^2 + 4$$

$\frac{P}{Q} = \frac{\pm 4, \pm 2, \pm 1}{\pm 1}$

$\pm 4, \pm 2, \pm 1$

List the possible zeros for

$$f(x) = 15x^3 + 14x^2 - 3x - 2$$

$$\frac{P}{Q} = \frac{\pm 1, \pm 2}{\pm 1, \pm 3, \pm 5, \pm 15}$$

$\pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{1}{15}, \pm \frac{2}{15}$

Find all rational zeros:

$$f(x) = x^3 + 2x^2 - 5x - 6$$

$$\frac{P}{Q} = \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1}$$

$\pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ & & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & \boxed{0} \end{array}$$

$(x+1)(x^2+x-6)=0$

$(x+1)(x-2)(x+3)=0$

$x = -1, 2, -3$

Solve the equation:

$$x^4 - 6x^2 - 8x + 24 = 0$$

$$\frac{P}{Q} = \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24}{\pm 1}$$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -6 & -8 & 24 \\ & & 2 & 4 & -4 & -24 \\ \hline 2 & 1 & 2 & -2 & -12 & \boxed{0} \\ & & 2 & 8 & 12 & \\ \hline & 1 & 4 & 6 & \boxed{0} & \end{array}$$

$$(x-2)(x-2)(x^2+4x+6)$$

$$x = \frac{-4 \pm \sqrt{16 - 4(1)(6)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{-8}}{2} = \frac{-4 \pm 2\sqrt{2}\,i}{2}$$

$$\boxed{x = 2, 2 \quad \boxed{-2 \pm \sqrt{2}\,i}}$$

Properties of Polynomial Equations:

- a polynomial of degree n has n roots.
- if a+bi is a root then a-bi is also a root.

$7i \quad -7i$

Fundamental Theorem of Algebra:

If f(x) is a polynomial of degree n and n>=1, then f(x)=0 has at least one complex root.

Factor: $x^4 - 3x^2 - 28$

under rational numbers

$(x^2-7)(x^2+4)$

under real numbers

$(x+\sqrt{7})(x-\sqrt{7})(x^2+4)$ $x^2 - -4$

under complex numbers

$(x+\sqrt{7})(x-\sqrt{7})(x+2i)(x-2i)$

Linear Factorization:

if c,d,e,f,.... are complex numbers

f(x)=a_n(x-c)(x-d)(x-e)(x-f)....

x^2-1 over rational

$(x-1)(x+1)$ $= x = \pm 1$

x^2-3 over real

$(x+\sqrt{3})(x-\sqrt{3})$

x^2+1 over complex

$(x^2-(-1))$

$(x+i)(x-i)$

Find a forth degree polynomial with real coefficients that has zeros -2,2,i. Such that f(3)=-150

$f(x) = a(x+2)(x-2)(x-i)(x+i)$

$f(x) = a(x^2-4)(x^2+1)$

$f(x) = a(x^4-3x^2-4)$

$$\frac{-150}{3^4-3(3)^2-4} = \frac{a(3^4-3(3)^2-4)}{(3^4-3(3)^2-4)}$$

$-3 = a$

$\boxed{f(x) = -3x^4+9x^2+12}$

Descartes Rule of Signs:

The number of positive real zeros is equal to the number of sign changes in f(x) or less than by an even integer.

The number of negative real zeros is equal to the number of sign changes in f(-x) or less than by an even integer.

Find the possible number of positive and negative real zeros.

$$f(x) = x^3 + 2x^2 + 5x + 4$$

$+\mathbb{R}$ = no $+\mathbb{R}$ zeros

$-\mathbb{R}$ $f(-x) = (-x)^3 + 2(-x^2) + 5(-x) + 4$

$\underbrace{-x^3 + 2x^2 - 5x + 4}$

$-\mathbb{R} = 3, 1$

$+\mathbb{R}$	$-\mathbb{R}$	I	T
0	3	0	3
0	1	2	3

$f(x) = 3x^5 + 3x^4 - 2x^2 + 3x - 4$

$+\mathbb{R} \to f(x) \to 3, 1$

$-\mathbb{R} \to f(-x) \to 2, 0$

$3(-x)^5 + 3(x)^4 - 2(-x)^2 + 3(-x) - 4$

$-3x^5 + 3x^4 - 2x^2 - 3x - 4$

$+\mathbb{R}$	$-\mathbb{R}$	I	T
3	2	0	5
3	0	2	5
1	2	2	5
1	0	4	5

Suggested Homework:
Chapter 2.5 pg. 302 #'s
5,11,17,25,31,33,39,43,53